Math 2001: Negation II (Katherine Stange, Spring 2018)

Task 1: Mark if each statement is true or false.
Task 2: Negate each statement.
Task 3 (sanity check): Mark if the negation is true or false.

1. There exists a negative integer which is a perfect square (F).

   Some possible negations:

   - There does not exist a negative integer which is a perfect square (T).
   - No negative integer is a perfect square (T).
   - Every negative integer is not a perfect square (T).

2. There exists a real number x such that $x = -x$ (T).

   Some possible negations:

   - There does not exist a real number x such that $x = -x$ (F).
   - No real number x satisfies $x = -x$ (F).
   - All real numbers x satisfy $x \neq -x$ (F).

3. Every integer is a rational number (T).

   Some possible negations:

   - There is an integer which is not a rational number (F).
   - There is an integer which is irrational (F).

4. All real numbers are either positive or negative (F).

   Some possible negations:

   - Not all real numbers are either positive or negative (T).
   - There are real numbers which are neither positive nor negative (T).
   - There are real numbers which are non-positive and non-negative (T).

5. If $x \in \mathbb{Z}$ is even, then $x > 3$ (F).

   Some possible negations:

   - There is some $x \in \mathbb{Z}$ which is even and satisfies $x \leq 3$ (T).
   - There is some $x \in \mathbb{Z}$ which is even but satisfies $x \leq 3$ (T).

6. If $x \in \mathbb{Z}$ is even, then x^2 is even (T).

   Some possible negations:

   - There is some $x \in \mathbb{Z}$ which is even but x^2 is odd (F).
   - There is some $x \in \mathbb{Z}$ which is even and x^2 is odd (F).

7. If $x \in \mathbb{Z}$ is a negative perfect square, then x is prime (T).

   Some possible negations:

   - There is some $x \in \mathbb{Z}$ which is a negative perfect square and not prime (F).
   - There is some $x \in \mathbb{Z}$ which is a negative perfect square but not prime (F).

8. All integers are rational and positive (F).

   Some possible negations:

   - There is some integer which is irrational or non-positive (T).
   - There is some integer which is either irrational or non-positive (T).
   - There exists an integer which is either irrational or non-positive (T).

9. All integers are rational and real (T).

   Some possible negations:

   - There exists an integer which is either irrational or non-real (F).
   - There is an integer which is either irrational or non-real (F).

10. Every integer is either prime or even (F).

    Some possible negations:

    - There is an integer which is neither prime nor even (T).
    - There is an integer which is not prime and not even (T).

11. Every integer greater than 5 is either odd or composite (T).

    Some possible negations:

    - There is some integer greater than 5 which is neither odd nor composite (F).
    - There is some integer greater than 5 which is even and not composite (F).