

$$143 = 13 \cdot 11 \quad (\text{Dec } 11)$$

$$\text{Solve } x^2 \equiv 3 \pmod{143} \iff \left\{ \begin{array}{l} x^2 \equiv 3 \pmod{13} \\ x^2 \equiv 3 \pmod{11} \end{array} \right\}$$

$$x^2 \equiv 3 \pmod{13}$$

$$4^2 \equiv 16 \equiv 3$$

$$x \equiv 4, -4$$

$$x^2 \equiv 3 \pmod{11}$$

$$5^2 \equiv 25 \equiv 3 \pmod{11}$$

$$x \equiv 5, -5$$

CRT. Combine:

$$\begin{array}{l} x \equiv 4 \pmod{13} \\ x \equiv 5 \pmod{11} \end{array}$$

$$\hline x \equiv 82$$

$$\begin{array}{l} x \equiv 4 \pmod{13} \\ x \equiv -5 \pmod{11} \end{array}$$

$$\hline x \equiv 17$$

$$\begin{array}{l} x \equiv -4 \pmod{13} \\ x \equiv 5 \pmod{11} \end{array}$$

$$\hline x \equiv 126$$

$$\begin{array}{l} x \equiv -4 \pmod{13} \\ x \equiv -5 \pmod{11} \end{array}$$

$$\hline x \equiv 61$$

4 solutions

$$x^2 \equiv 33 \pmod{143}$$

$$\iff \left\{ \begin{array}{l} x^2 \equiv 7 \pmod{13} \\ x^2 \equiv 0 \pmod{11} \end{array} \right\}$$

$$x^2 \equiv 7 \pmod{13}$$

$$1^2 \equiv 1 \quad 2^2 \equiv 4 \quad 3^2 \equiv 9 \quad 4^2 \equiv 3$$

$$5^2 \equiv 12 \quad 6^2 \equiv 10$$

$$7^2 \equiv (-6)^2 \equiv 10 \quad 8^2 \equiv (-5)^2 \equiv 12$$

no square roots.

$\Rightarrow$ no solution.

1 solution. $x^2 \equiv 0 \pmod{143} \Leftrightarrow \begin{cases} x^2 \equiv 0 \pmod{13} \\ x^2 \equiv 0 \pmod{11} \end{cases} \Leftrightarrow \begin{cases} x \equiv 0 \pmod{13} \\ x \equiv 0 \pmod{11} \end{cases} \Leftrightarrow x \equiv 0 \pmod{143}.$

2 solutions? $x^2 \equiv 66 \pmod{143} \Leftrightarrow \begin{cases} x^2 \equiv 1 \pmod{13} \\ x^2 \equiv 0 \pmod{11} \end{cases} \Leftrightarrow \begin{cases} x \equiv \pm 1 \pmod{13} \\ x \equiv 0 \pmod{11} \end{cases}$

CRT. $\begin{cases} x \equiv 1 \pmod{13} \\ x \equiv 0 \pmod{11} \end{cases}$

 $x \equiv 66 \pmod{143}$

$\begin{cases} x \equiv -1 \pmod{13} \\ x \equiv 0 \pmod{11} \end{cases}$

 $x \equiv 77 \pmod{143}$