Solutions to Daily Due Oct 23 (AI generated)

Math 4440/5440, Coding and Cryptography, Fall 2026

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(all computations computer-verified)

Problem 1 was the online complex numbers worksheet, which has its own built-in feedback. Throughout, a qubit $a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$ gives outcome 0 with probability $|a|^2$ and outcome 1 with probability $|b|^2$. To measure in a different basis, first rewrite the state in that basis, then read off the squared lengths of the coefficients.

Problem 2: the length of $\frac{3}{5} + \frac{4}{5}i$

The length of $a + bi$ is $\sqrt{a^2 + b^2}$, so

$$\left|\tfrac{3}{5} + \tfrac{4}{5}i\right| = \sqrt{\tfrac{9}{25} + \tfrac{16}{25}} = \sqrt{\tfrac{25}{25}} = 1.$$

This is the 3-4-5 right triangle scaled down by 5. A common slip is to compute $|a+bi|^2 = (a+bi)^2$; the right formula is $|a + bi|^2 = (a + bi)\overline{(a + bi)} = (a + bi)(a - bi) = a^2 + b^2$.

Problem 3: the qubit $i|0\rangle + \sqrt{3}|1\rangle$

(1) Normalize. The squared lengths of the coefficients are $|i|^2 = 1$ and $|\sqrt{3}|^2 = 3$, so $|a|^2+|b|^2 = 4$ and we divide by $\sqrt{4} = 2$:

$$\frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle.$$

Check: $\left|\frac{i}{2}\right|^2 + \left|\frac{\sqrt{3}}{2}\right|^2 = \frac{1}{4} + \frac{3}{4} = 1$. (Note that $|i|^2 = 1$, not $i^2 = -1$: lengths are always nonnegative real numbers.)

(2) Measure. Outcome 0 with probability $\left|\frac{i}{2}\right|^2 = \frac{1}{4}$, and outcome 1 with probability $\left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4}$. The factor i on $|0\rangle$ has no effect on the probabilities; it is a *phase*, and phases only become visible when states are added together (as in the next problems).

Problem 4: the V basis measured in the L basis

Recall $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle)$ and $|\nwarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\rightarrow\rangle)$.

(1) $|\nearrow\rangle$ in the L basis. It is already written in the L basis, with coefficients $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$. So

$$|\uparrow\rangle \text{ with probability } \tfrac{1}{2}, \qquad |\rightarrow\rangle \text{ with probability } \tfrac{1}{2}.$$

This is the "diagonal photon through a vertical filter" situation: a coin flip.

(2) $|\nearrow\rangle + i|\nwarrow\rangle$ in the L basis. First expand in the L basis:

$$|\nearrow\rangle + i|\nwarrow\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\rightarrow\rangle\right) + \tfrac{i}{\sqrt{2}}\left(|\uparrow\rangle - |\rightarrow\rangle\right) = \tfrac{1+i}{\sqrt{2}}|\uparrow\rangle + \tfrac{1-i}{\sqrt{2}}|\rightarrow\rangle.$$

Now normalize. $|1+i|^2 = 1^2 + 1^2 = 2$ and $|1-i|^2 = 2$, so the squared lengths of the coefficients are $\frac{2}{2} + \frac{2}{2} = 2$, and we divide by $\sqrt{2}$:

$$\frac{1+i}{2}|\uparrow\rangle + \frac{1-i}{2}|\rightarrow\rangle.$$

(Equivalently, normalize first: $|\nearrow\rangle + i|\nwarrow\rangle$ has squared length $1 + |i|^2 = 2$ since $|\nearrow\rangle$ and $|\nwarrow\rangle$ are orthonormal, so the normalized state is $\frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{i}{\sqrt{2}}|\nwarrow\rangle$. Expanding gives the same answer.)

Measuring in the L basis:

$$|\uparrow\rangle \text{ with probability } \left|\tfrac{1+i}{2}\right|^2 = \tfrac{2}{4} = \tfrac{1}{2}, \qquad |\rightarrow\rangle \text{ with probability } \left|\tfrac{1-i}{2}\right|^2 = \tfrac{1}{2}.$$

Problem 5: $|\psi\rangle = \frac{1}{\sqrt{5}}|\uparrow\rangle + \frac{2}{\sqrt{5}}|\rightarrow\rangle$

(1) In the L basis. Read off the coefficients:

$$|\uparrow\rangle \text{ with probability } \tfrac{1}{5}, \qquad |\rightarrow\rangle \text{ with probability } \tfrac{4}{5}.$$

(2) In the V basis. Invert the definitions of $|\nearrow\rangle$ and $|\nwarrow\rangle$ (add and subtract them):

$$|\uparrow\rangle = \tfrac{1}{\sqrt{2}}\left(|\nearrow\rangle + |\nwarrow\rangle\right), \qquad |\rightarrow\rangle = \tfrac{1}{\sqrt{2}}\left(|\nearrow\rangle - |\nwarrow\rangle\right).$$

Substitute:

$$|\psi\rangle = \tfrac{1}{\sqrt{5}}\cdot\tfrac{1}{\sqrt{2}}\left(|\nearrow\rangle + |\nwarrow\rangle\right) + \tfrac{2}{\sqrt{5}}\cdot\tfrac{1}{\sqrt{2}}\left(|\nearrow\rangle - |\nwarrow\rangle\right) = \tfrac{3}{\sqrt{10}}|\nearrow\rangle - \tfrac{1}{\sqrt{10}}|\nwarrow\rangle.$$

This is already normalized: $\frac{9}{10} + \frac{1}{10} = 1$ (changing basis never changes the length). So

$$|\nearrow\rangle \text{ with probability } \tfrac{9}{10}, \qquad |\nwarrow\rangle \text{ with probability } \tfrac{1}{10}.$$

Notice the minus sign mattered: the $|\nwarrow\rangle$ coefficient is $\frac{1}{\sqrt{10}}(1-2)$, a cancellation, while the $|\nearrow\rangle$ coefficient is $\frac{1}{\sqrt{10}}(1+2)$, a reinforcement. Same state, two different bases, two different sets of probabilities.