Solutions to Daily Due Oct 30 (AI generated)

Math 4440/5440, Coding and Cryptography, Fall 2026

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(all computations computer-verified)

Write $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ *and* $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ *for short.*

Problem 1: two qubits, measured separately

Since the two qubits are prepared and measured independently, the probability of a pair of outcomes is the product of the individual probabilities.

(1) First qubit $|0\rangle$ (outcome 0 with probability 1), second $|+\rangle$ (outcomes 0, 1 each with probability $\frac{1}{2}$):

$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$\frac{1}{2}$	$\frac{1}{2}$	0	0

(2) Both $|+\rangle$: each pair has probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(3) First qubit $\frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$ (outcome 0 with probability $\frac{1}{5}$, outcome 1 with probability $\frac{4}{5}$), second $|-\rangle$ (each outcome $\frac{1}{2}$):

$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$\frac{1}{5} \cdot \frac{1}{2} = \frac{1}{10}$	$\frac{1}{10}$	$\frac{4}{5} \cdot \frac{1}{2} = \frac{2}{5}$	$\frac{2}{5}$

The minus sign in $|-\rangle$ plays no role here, since only $\left|\pm\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$ enters.

Problem 2: two qubits as one state

(1) $|0\rangle|+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$, whose squared coefficients are $\frac{1}{2}, \frac{1}{2}, 0, 0$ on $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. This matches 1(1).

(2) Multiply out, keeping the first qubit first:

$$|+\rangle|+\rangle = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle.$$

Each squared coefficient is $\frac{1}{4}$, matching 1(2).

(3)

$$\left(\tfrac{1}{\sqrt{5}}|0\rangle + \tfrac{2}{\sqrt{5}}|1\rangle\right)\left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\right) = \tfrac{1}{\sqrt{10}}|00\rangle - \tfrac{1}{\sqrt{10}}|01\rangle + \tfrac{2}{\sqrt{10}}|10\rangle - \tfrac{2}{\sqrt{10}}|11\rangle.$$

Squared coefficients: $\frac{1}{10}, \frac{1}{10}, \frac{4}{10}, \frac{4}{10}$, matching 1(3). The minus signs are on $|01\rangle$ and $|11\rangle$, the two outcomes where the second bit is 1.

(4) The totals: $\frac{1}{2} + \frac{1}{2} = 1$; $4 \cdot \frac{1}{4} = 1$; $\frac{1}{10} + \frac{1}{10} + \frac{4}{10} + \frac{4}{10} = 1$. In general, if $|a|^2 + |b|^2 = 1$ and $|c|^2 + |d|^2 = 1$, then the product state has squared coefficients $|a|^2|c|^2 + |a|^2|d|^2 + |b|^2|c|^2 + |b|^2|d|^2 = (|a|^2 + |b|^2)(|c|^2 + |d|^2) = 1$, so a product of normalized states is always normalized.

Problem 3: the state $|\Phi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

(1) The coefficients are $\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}$, so

00	01	10	11
$\frac{1}{2}$	0	0	$\frac{1}{2}$

The two bits always agree.

(2) $\Pr(\text{first bit} = 0) = \Pr(00) + \Pr(01) = \frac{1}{2} + 0 = \frac{1}{2}$.

(3) If the first qubit came out 0, the only outcome consistent with that is 00 (the outcome 01 has probability 0). So Bob knows the second qubit will certainly give 0, before he measures it. (In the language of lecture: measuring the first qubit as 0 collapses the state to $|00\rangle$.) Compare $|+\rangle|+\rangle$ from 2(2): there, after seeing a 0 in the first qubit, the second is still 0 or 1 with probability $\frac{1}{2}$ each, exactly as before. Knowing the first outcome told Bob nothing.

(4) Suppose $|\Phi\rangle = (x|0\rangle + y|1\rangle)(z|0\rangle + w|1\rangle) = xz|00\rangle + xw|01\rangle + yz|10\rangle + yw|11\rangle$. Comparing coefficients:

$$xz = \tfrac{1}{\sqrt{2}}, \qquad xw = 0, \qquad yz = 0, \qquad yw = \tfrac{1}{\sqrt{2}}.$$

From $xw = 0$, either $x = 0$ or $w = 0$. But $x = 0$ contradicts $xz \neq 0$, and $w = 0$ contradicts $yw \neq 0$. So no such x, y, z, w exist: $|\Phi\rangle$ is not a product state. It is **entangled**. (A slicker way to say it: for a product state the 2×2 array of coefficients $\begin{pmatrix} xz & xw \\ yz & yw \end{pmatrix}$ has determinant $xz \cdot yw - xw \cdot yz = 0$, while for $|\Phi\rangle$ it is $\frac{1}{2} - 0 \neq 0$.)

Problem 4: normalize first

The squared lengths of the coefficients of $|00\rangle + 2i|01\rangle - 2|11\rangle$ are $1 + |2i|^2 + |-2|^2 = 1 + 4 + 4 = 9$, so divide by 3:

$$\tfrac{1}{3}|00\rangle + \tfrac{2i}{3}|01\rangle - \tfrac{2}{3}|11\rangle.$$

Measuring both qubits:

00	01	10	11
$\frac{1}{9}$	$\frac{4}{9}$	0	$\frac{4}{9}$

(Check: $\frac{1+4+0+4}{9} = 1$.)

Product or entangled? Try $xz = \frac{1}{3}$, $xw = \frac{2i}{3}$, $yz = 0$, $yw = -\frac{2}{3}$. From $yz = 0$: if $y = 0$ then $yw = 0$, contradicting $yw = -\frac{2}{3}$; if $z = 0$ then $xz = 0$, contradicting $xz = \frac{1}{3}$. So it is **entangled**. (Determinant check: $xz \cdot yw - xw \cdot yz = -\frac{2}{9} - 0 \neq 0$.) The telltale sign in the probability table is a single 0 sitting among three nonzero entries: in a product state, the probability of 10 is $\Pr(\text{first} = 1)\Pr(\text{second} = 0)$, and both of those factors are nonzero here ($\Pr(\text{first} = 1) = \frac{4}{9}$ from the 11 outcome, $\Pr(\text{second} = 0) = \frac{1}{9}$ from the 00 outcome).